FREE FORMULA SHEET

AQA A Level Maths Formula Sheet

PURE · STATISTICS · MECHANICS

AQA A Level Mathematics (7357) · Pure, Statistics and Mechanics

1 Given in Your Exam

These formulas are in the AQA "Formulae and Statistical Tables" booklet given out in every paper. You don't need to memorise them, but you do need to know when and how to use each one. It also includes statistical tables for the binomial and normal distributions, although most students use their calculator for these.

PURE

Binomial Series

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

For positive whole numbers n

General Binomial Expansion

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots$$

Valid for $|x| < 1$ when n is negative or a fraction

Arithmetic Series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n-1)d]$$

a is the first term, l the last term, d the common difference

Geometric Series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

a is the first term, r the common ratio

Small Angle Approximations

$$\sin\theta \approx \theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan\theta \approx \theta$$

θ in radians

Compound Angle Formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Derivatives of Trig Functions

$$\tan x \to \sec^2 x$$

$$\sec x \to \sec x \tan x$$

$$\cot x \to -\operatorname{cosec}^2 x$$

$$\operatorname{cosec} x \to -\operatorname{cosec} x \cot x$$

f(x) → f′(x)

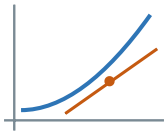
Product and Chain Rules

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Quotient Rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$



Differentiation From First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration by Parts

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

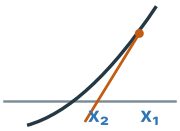
Integrating f′(x) ÷ f(x)

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

Integrals of tan x and cot x

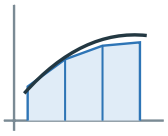
$$\int \tan x\,dx = \ln|\sec x| + c$$

$$\int \cot x\,dx = \ln|\sin x| + c$$



Newton–Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

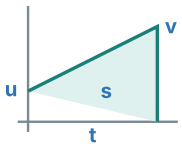


The Trapezium Rule

$$\int_a^b y\,dx \approx \frac{h}{2}\{(y_0 + y_n) + 2(y_1 + \ldots + y_{n-1})\}$$

$$h = \frac{b-a}{n}$$

MECHANICS



Constant Acceleration (suvat)

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

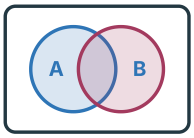
$$s = vt - \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

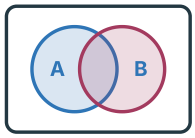
Motion in a straight line with constant acceleration

STATISTICS



Probability: A or B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Probability: A and B

$$P(A \cap B) = P(A) \times P(B|A)$$

Standard Deviation

$$\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

The Binomial Distribution

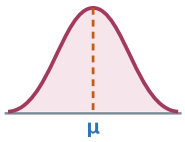
If $X \sim B(n, p)$:

$$P(X = x) = \binom{n}{x}p^x(1-p)^{n-x}$$

$$\text{Mean} = np$$

$$\text{Variance} = np(1-p)$$

Sample Mean From a Normal Distribution



$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

Used in hypothesis tests for the mean

2 Learn These by Heart

These are **not** given in the exam. Students lose marks on them every year, so learn them until you can write them from memory.

ALGEBRA AND FUNCTIONS

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of Indices

$$a^x \times a^y = a^{x+y}$$

$$a^x \div a^y = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

Laws of Logarithms

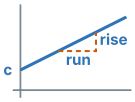
$$x = a^n \Leftrightarrow n = \log_a x$$

$$\log_a x + \log_a y = \log_a(xy)$$

$$\log_a x - \log_a y = \log_a \frac{x}{y}$$

$$k \log_a x = \log_a(x^k)$$

$a > 0, x > 0$



Straight Lines

$$y - y_1 = m(x - x_1)$$

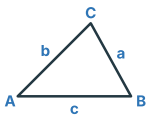
Perpendicular: $m_1 m_2 = -1$

nth Terms of Sequences

Arithmetic: $u_n = a + (n - 1)d$

Geometric: $u_n = ar^{n-1}$

TRIGONOMETRY



Sine Rule, Cosine Rule and Area

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area} = \frac{1}{2}ab \sin C$$

Pythagorean Identities

$$\cos^2 A + \sin^2 A \equiv 1$$

$$\sec^2 A \equiv 1 + \tan^2 A$$

$$\text{cosec}^2 A \equiv 1 + \cot^2 A$$

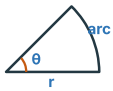
Double Angle Formulae

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

$\cos 2A$ can also be written as $2\cos^2 A - 1$ or $1 - 2\sin^2 A$



Arc Length and Sector Area

$$s = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

θ in radians

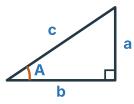
MENSURATION

Circles

$C = 2\pi r = \pi d$

$A = \pi r^2$

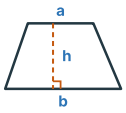
Pythagoras' Theorem



$c^2 = a^2 + b^2$

c is the hypotenuse

Trapezium and Prism



Area of a trapezium = $\frac{1}{2}(a + b)h$

Volume of a prism = area of cross-section × length

CALCULUS AND VECTORS

Standard Derivatives

$x^n \to nx^{n-1}$

$\sin kx \to k \cos kx$

$\cos kx \to -k \sin kx$

$e^{kx} \to ke^{kx}$

$\ln x \to \frac{1}{x}$

f(x) → f′(x)

Product and Chain Rules

$(fg)' = f'g + fg'$

$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Standard Integrals

$x^n \to \frac{x^{n+1}}{n+1}$ $(n \neq -1)$

$\cos kx \to \frac{1}{k}\sin kx$

$\sin kx \to -\frac{1}{k}\cos kx$

$e^{kx} \to \frac{1}{k}e^{kx}$

$\frac{1}{x} \to \ln|x|$

Add + c to each

Reverse Chain Rule and Area

$\int f'(g(x))g'(x)\,dx = f(g(x)) + c$

Area under a curve = $\int_a^b y\,dx$ $(y \geq 0)$

Magnitude of a Vector

$|xi + yj + zk| = \sqrt{x^2 + y^2 + z^2}$

MECHANICS

Forces and Newton's Second Law

Weight = mg

Friction: $F \leq \mu R$

$F = ma$

Use $g = 9.8\ m\ s^{-2}$ unless told otherwise

Variable Acceleration

$v = \frac{dr}{dt}$

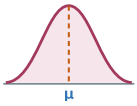
$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$

$r = \int v\,dt$

$v = \int a\,dt$

Mean

$$\bar{x} = \frac{\Sigma x}{n} = \frac{\Sigma fx}{\Sigma f}$$



Standardising a Normal Variable

$$Z = \frac{X - \mu}{\sigma}, \text{ where } X \sim N(\mu, \sigma^2)$$

Write It First

Write the formula down before you substitute. You can earn a method mark even if the answer goes wrong.

Check the Units

Make sure every length is in the same unit before you use a formula.

Test Yourself

Cover the right-hand side of each card and try to write the formula from memory.

Need Help Using These Formulas?

Book a free 90-minute lesson and I'll show your child exactly when and how to use each one.

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